Homework #4

ST 554

PMF 1

Suppose that $P(X = x) = 1/5$ for $x = 1, 2, 3, 4, 5$ and 0 elsewhere, is the pmf for a discrete random variable X.

a. Find $E[X]$
b. Find $E[X^2]$
c. Use these results to find $E[(X + 2)^2]$.
d. Find $V[X]$

PMF 2

Suppose X has a pmf which is positive at $x = -1, 0, 1$ and 0 elsewhere.

a. If $P(X = 0) = 1/4$, find $E[X^2]$.
b. If $P(X = 0) = 1/4$ and $E[X] = 1/4$, find $P(X = 1)$ and $P(X = -1)$.

Insurance Premium

An insurance company writes a policy to the effect that an amount of money A must be paid if some event E occurs within a year. If the company estimates that E will occur within a year with probability p, what premium c should the company charge the customer in order that the expected profit will be 10 percent of A? Note: the company will receive c from the customer and will pay out A only if event E occur (and profit can be negative).

The Gambler

A gambling book recommends the following "winning strategy" for a game of roulette. It recommends that a gambler bet $1 on *red*. If *red* appears (which has probability 18/38), then the gambler should take her $1 profit and quit. If the gambler loses this bet (which has probability 20/38), she should make additional $1 bets on red on each of the next two spins of the roulette wheel and then quit. Let X denote the gambler's winnings when she quits.

a. Find $P(X > 0)$
b. Are you convinced that the strategy is indeed a "winning" strategy? Explain your answer.
c. Find $E[X]$.

Students on Buses

A total of 4 buses carrying 148 students from the same school arrives at a football stadium. The buses carry, respectively, 40, 33, 25, and 50 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying this randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on her bus.

a. Which of $E[X]$ or $E[Y]$ do you think is larger? Why?
b. Compute $E[X]$ and $E[Y]$
c. What is the probability that the selected student and bus driver are from the same bus?
d. If two students are randomly selected, what is the probability they are from the same bus?
e. Suppose all 148 students are paired up (74 pairs). What is the expected number of pairs of students that are from the same bus?

Best-of-Seven Playoffs

Suppose two *equally matched* teams are playing a best of seven playoff (first team to win 4 games is the winner). Assume independent games. Find the expected number of games played. Hint: There are two ways (since two teams) for the playoffs to last k games.

Blitzstein & Hwang: Chapter 4

1, 7, 17, 18, 22, 29, 35

Extra problems: Chapter 4 (optional)

- solved: 21, 30, 31, 33
- interest: 9, 15, 16, 28, 49, 82

Computer Experiment: Simulate Stock Market (optional)

Let $Y_1, Y_2, \ldots$ be independent random variables such that $\Pr(Y_i = 1) = \Pr(Y_i = -1) = 1/2$. Let $X_n = \sum_{i=1}^{n} Y_i$. Think of $Y_i = 1$ as "the stock price increased by one dollar", $Y_i = -1$ as "the stock price decreased by one dollar", and X_n as the value of the stock on day n.

a. Find $E[X_n]$ and $V[X_n]$.
b. Simulate X_n and plot X_n versus n for $n = 1, 2, \ldots, 10^4$. Repeat the whole simulation several times. Notice two things. First, it's easy to *see* patterns in the sequence even though it is random (no real patterns). Second, you will find that the different runs look very different even though they were generated the same way. How do the calculations in part a explain this?
c. What should happen if we replaced X_n with $\bar{X}_n = X_n/n$?